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Predictably Unpredictable: A Look at Autocorrelation in Market Variables

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Abstract
Successful systematic trading strategies depend on some element of predictability of market behavior, however small. When trading futures, signals seek to predict market returns, while predictions of market volatility and correlations are made to allow for risk management of the portfolio. In this note we take a brief look at how easily each of these market variables is predicted and what implications this has for systematic trading.

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1. Introduction

Financial markets are largely unpredictable. For any given market, for example, it is not possible to say with any degree of certainty if this market will move up or down tomorrow, in the coming days or weeks. Yet there is *alpha* embedded in past market data that can be exploited when trading a large number of markets, where even a small edge (see Bethke and Tricker (2019)) in individual predictability aggregates to give a succesful trading strategy via *diversification* (see Tricker and Mitchell (2017)). While market returns are difficult to predict, volatility and correlation tend to be ‘stickier’, meaning that they do not change as rapidly. This is an advantage when trying to predict their levels in the future. The technical term for ‘stickiness’ is autocorrelation, which, as the name suggests, measures the correlation between values of the same variable at different time lags. When using a one-day lag, for example, it is the correlation between values of the same variable on consecutive days.

2. A Short Excursion

While returns are easily observed, the ‘true’ volatility or correlation are not, and usually a trailing sample estimate is used for both of these. Volatility is typically calculated as a trailing standard deviation of returns and correlation as a sample correlation on the same lookback. These trailing estimates are inherently smoothed, making them sticky by construction. To avoid this, and as we are trying to compare predictability of volatility and correlation to the predictability of returns, we proxy volatility by squared returns and correlation by cross-product of returns.

Before examining the autocorrelation of returns, squared returns and cross-product of returns for a range of futures markets, we set the scene and derive some theoretical results for independent, identically-distributed (IID) returns. Normally distributed IID returns are often used to model financial markets, offering theoretical tractability, despite shortcomings such as the absence of fat tails. The results we establish will serve as a benchmark with respect to which we can better interpret our later findings.

Let $X_t \sim N(0,1)$ be a random variable that represents our return at time t. Similarly, we have a return $X_{t+s} \sim N(0,1)$ at a later time $t+s$. In our model, the two returns are independent, and each has a mean and a volatility equal to zero and one, respectively. Their covariance is then given by

$$\mathrm{Cov}(X_t, X_{t+s}) = \mathbb{E}(X_t X_{t+s}) = \mathbb{E}(X_t)\mathbb{E}(X_{t+s}) = 0.$$

As anticipated, the covariance (and thus correlation[1]) is zero. What about squared returns? The squared returns in our model do not have a mean of zero. Instead, we find that

$$\mathbb{E}(X_t^2) = \mathbb{E}(X_{t+s}^2) = \mathrm{Var}(X_t) = \mathrm{Var}(X_{t+s}) = 1,$$

which allows us to calculate

$$\begin{aligned}\mathrm{Cov}(X_t^2, X_{t+s}^2) &= \mathbb{E}((X_t^2 - 1)(X_{t+s}^2 - 1)) \\ &= \mathbb{E}(X_t^2 X_{t+s}^2 - X_t^2 - X_{t+s}^2 + 1) \\ &= \mathbb{E}(X_t^2)\mathbb{E}(X_{t+s}^2) - \mathbb{E}(X_t^2) - \mathbb{E}(X_{t+s}^2) + 1 \\ &= 0.\end{aligned}$$

Again, for IID returns, the squared returns also display zero autocorrelation.

3. Autocorrelation of Returns, Volatility and Correlation

We are now ready to calculate time series of returns, squared returns and cross-products of returns for a range of financial futures markets and determine the autocorrelation for each. The results are aggregated across markets and presented in Figure 1. We find that autocorrelation is extremely low if at all present for consecutive one-day returns, while it is much higher for squared returns or cross-product of returns. For instance, at 5 days, autocorrelation is 0 for returns, while it is 18% for squared returns, and 10% for return cross-products. There is also a clear pattern of larger autocorrelation at smaller lags for squared returns and

[1] The correlation ρ between two random variables X and Y with volatilities σ_X and σ_Y is defined as $\rho = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}$. When $\sigma_X = \sigma_Y = 1$ we thus have $\rho = \mathrm{Cov}(X, Y)$.

return cross-products. This phenomenon, which does not exist for IID returns, is termed 'volatility clustering' in the context of squared returns and/or volatility, meaning that volatile periods are not spread evenly in time, but rather take turns with calmer periods. Or, to cite Mandelbrot who first observed the phenomenon, see Mandelbrot (1963), "large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes". It has since been documented and analyzed by a variety of researchers, such as Ding et al. (1993) and Cont (2001), just to name some prominent examples.

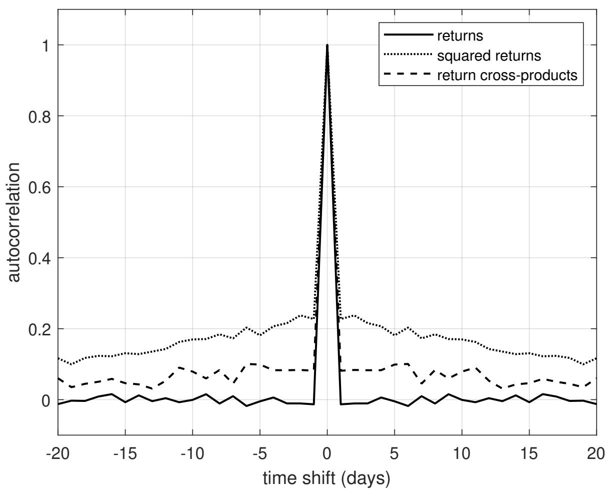

Figure 1. Autocorrelation of returns, squared returns and return cross-products at different lags and aggregated across markets.

4. Predicting Returns, Volatility and Correlation

Variables that exhibit higher autocorrelation tend to stay closer to their previous values than variables that do not. We can illustrate how this impacts predictability by measuring the coefficient of determination, R^2, in a regression for each of our variables. High values of R^2 indicate a strong (linear) relationship between the variables that are being regressed, with higher R^2 implying greater predictability. For each time point, we regress the average over n days of the given variable in the future, on the average over n days of that variable in the past.

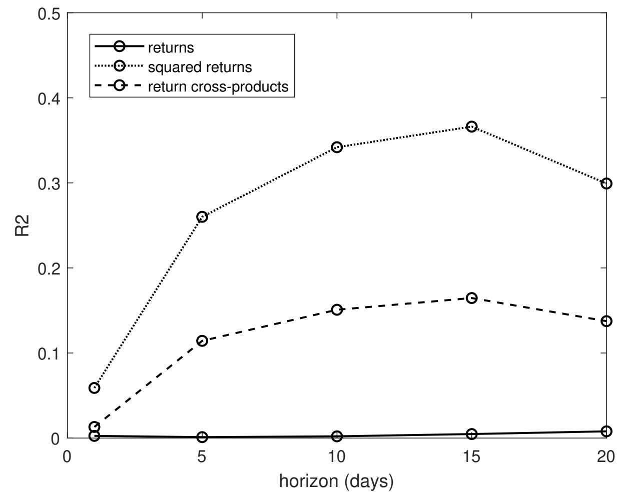

Figure 2. R^2 for prediction - realization regressions of returns, squared returns and return cross-products at different horizons and aggregated across markets.

5. Conclusion

Despite the unpredictability of financial markets, we find that - fortunately - some predictability remains. Systematic trading models predict returns to build trading signals, and attempt to harness whatever alpha they can find. Due to the low autocorrelation of returns, this is a challenging problem. Volatilities and correlations are somewhat easier to predict because of their disctinctly non-zero autocorrelation. This can be leveraged in the management of risk of said systematic trading strategies.

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