Can You Trend-Follow Trend-Following?

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Abstract
If trend-following can successfully time asset returns, can we apply the strategy to its own NAV to boost performance? In this research note we introduce a simplified model that preserves the nature of generic trend-following strategies, reducing them to an analytically tractable form. We then deduce some mathematical characteristics from trend-following P&L returns that are distinct from asset returns, showing that the former is less amenable to being trend-followed again. Our results cast doubt upon the ability of an allocator to time strategies to enhance portfolio performance.

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1. Introduction

Many portfolio allocators do some form of strategy timing, ranging from basic periodic rebalancing to chasing / counteracting recent performance to analyzing the strategy's trading style in various macroeconomic regimes. After all, as active trading strategies expect to generate alpha by timing asset returns, it is natural to attempt to extend the timing to strategies themselves as tradables. Whereas active allocators manage strategies in a "long-only" manner, here we consider the full gamut of going "long-short". For the strategies we restrict our consideration to a set of simple trend-followers that are analytically tractable, applied to one underlying asset only.

Typically an asset's returns must exhibit positive autocorrelation for trend-following to work. The shape of the autocorrelation function typically decays to zero as the number of lagged returns increases. Time can therefore be discretized into equal-length periods, with the period corresponding to the half-life of the autocorrelation function. If r_t denotes asset returns for period t, then under our assumptions the lag-1 autocorrelation $\rho = \mathrm{cor}(r_t, r_{t-1})$ is significant, while $\mathrm{cor}(r_t, r_{t-i})$ for $i > 1$ decays exponentially.

We define a family of simple trend-following models, the n-period trend-follower taking positions $P_t^{(n)} = \sum_{i=0}^{n-1} r_{t-i}$ at the end of each period t. For $n = 1$ the model is adapted to the shape of the autocorrelation function and can be shown to give the optimal performance within the family, while higher n values give progressively slower models.

Compared with actual trend-following strategies, our construction represents simplifications in several areas: discretizations in time and positioning, and lack of multi-asset portfolio construction. Despite their simplicity, these models capture the essence of trend-following by being highly correlated with them, as well as having similar P&L autocorrelation structure, which we shall see affects their amenability to being timed further. For most macro asset classes, the discretized time period is a few months which corresponds well to a portfolio allocator's decision frequency. We now analyze our models in detail.

2. Analysis and Results

First, we state our assumptions and model construction:

1. Asset returns $r_t \sim N(0, 1)$, $\mathrm{cor}(r_t, r_{t-i}) = \rho^i$.
2. Fix n, define positions $P_t^{(n)} = \sum_{i=0}^{n-1} r_{t-i}$.
3. Strategy P&L is $R_t^{(n)} = P_{t-1}^{(n)} r_t = \sum_{i=1}^{n} r_{t-i} r_t$.

Proposition 1. *Generally, let $r_t \sim N(\mu, 1)$, $\mathrm{cor}(r_t, r_{t-i}) = \theta_i$. Then the information ratio of the n-period strategy is*

$$\mathrm{IR}(R_t^{(n)}) \approx \frac{\sum_{i=1}^{n} \theta_i}{\sqrt{n + \sum_{i=1}^{n-1} (n-i)\theta_i + \left(\sum_{i=1}^{n} \theta_i\right)^2}}.$$

Under the further assumptions of (1) above, $\mathrm{IR}(R_t^{(n)})$ is approximately $\rho/\sqrt{n}$.

The dependence of information ratio on the drift is μ^2 and dropped from the approximation. Higher asset return autocorrelation ρ leads to better trend-following performance. Also, $n = 1$ is optimal within this family of strategies.

Proposition 2. *The autocorrelations of strategy P&L are*

$$\mathrm{cor}\left(R_t^{(n)}, R_{t+s}^{(n)}\right) \approx \begin{cases} \frac{(n-s)\rho^s}{n} & \text{if } n > s, \\ \frac{2\rho^{s+1}}{n} & \text{if } n = s, \\ \frac{\rho^{s+1} + \rho^{2s-n+1}}{n} & \text{if } n < s. \end{cases}$$

In particular, for $n = 1$ the P&L autocorrelations are approximately $\left[2\rho^2, \rho^3, \dots\right]$ for $s \in \{1, 2, \dots\}$. If we apply trend-following to these P&L returns using the general form of Proposition 1, the expected information ratio is on the order of ρ^2, much smaller than ρ. [1] This shows that for the optimal trend-following model there is little room for improvement by trend-following it.

[1] Strictly speaking, $R_t^{(n)}$ do not have normal distributions, they have positive excess kurtosis. The numerator of IR does not require normality, while the denominator is a lower bound due to kurtosis. So we have an upper bound of the information ratio.

On the other hand, for $n > 1$ the P&L autocorrelations are approximately $\left[\frac{n-1}{n}\rho, \frac{n-2}{n}\rho^2, \dots\right]$. If we apply an m-period trend-follower to these P&L returns using Proposition 1, we can expect an information ratio of $\frac{n-1}{n\sqrt{m}}\rho$. While this does represent a potential to improve performance through trend timing (the potential being bigger for large n corresponding to trend-following models that are too slow to be practical), it is strictly less than ρ which is the information ratio of the optimal 1-period trend-follower on the asset returns.

In fact, any combination of n-period trend-follower together with any m-period trend-on-trend cannot match the 1-period trend-follower. This is a two tradable optimization problem with individual information ratios $\frac{\rho}{\sqrt{n}}$ and $\frac{n-1}{n\sqrt{m}}\rho$, and positive correlation between the two. The information ratio of the combined portfolio is capped at $\sqrt{\frac{1}{n} + \frac{(n-1)^2}{n^2 m}}\rho \leq \rho$, with equality only achieved when $m = n = 1$. In other words, rather than trend-following trend-following, one should just run the best trend-following.

3. Conclusion

For a trend-following strategy that is optimally tuned to autocorrelations of the traded asset, as one should expect from a capable manager, its P&L returns have distinct characteristics than asset returns by exhibiting autocorrelation an order of magnitude smaller, therefore making trend-following on trend-following less attractive. More over, the lack of P&L autocorrelation makes strategy returns look more like random walks with drift, which are inherently harder to be timed by any means, not only trend-following over again. In addition, our analysis can be applied similarly to timing mean-reversion strategies as well if we consider situations where $\rho < 0$.

4. Appendix: Proofs of Statements

We will make extensive use of

Theorem (Isserlis). *Let W, X, Y, Z be zero-mean random variables with normal distribution. We have $E(XYZ) = 0$ and*

$$
\begin{aligned}
E(WXYZ) &= E(WX)E(YZ) \\
&\quad + E(WY)E(XZ) \\
&\quad + E(WZ)E(XY).
\end{aligned}
$$

Proof of Proposition 1. Working in general with $r_t \sim N(\mu, 1)$ and $E(r_t, r_{t-i}) = \theta_i$, we have:

$$
\begin{aligned}
E(R_t^{(n)}) &= \sum_{i=1}^{n} \theta_i + n\mu^2 \approx \sum_{i=1}^{n} \theta_i, \\
E\left((R_t^{(n)})^2\right) &= \sum_{i=1}^{n}\sum_{j=1}^{n} E(r_{t-i} r_{t-j} r_t^2) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \left((1+\mu^2)\theta_{|i-j|} + 2\theta_i\theta_j + 2\mu^2(\theta_i + \theta_j) + \mu^4\right) \\
&\approx n + \sum_{i=1}^{n-1}(n-i)\theta_i + 2\left(\sum_{i=1}^{n}\theta_i\right)^2,
\end{aligned}
$$

where we have dropped all terms $O(\mu^2)$. Since

$$
\mathrm{IR}\left(R_t^{(n)}\right) = \frac{E(R_t^{(n)})}{\sqrt{E\left((R_t^{(n)})^2\right) - \left(E(R_t^{(n)})\right)^2}},
$$

the first part of Proposition 1 follows. When $\theta_i = \rho^i$, we have $\sum_{i=1}^{n} \rho^i = (\rho - \rho^{n+1})/(1-\rho)$. Substituting into the equations above, ignoring the terms that are $O(\rho^2)$ proves the second part. □

Proof of Proposition 2. We compute for $s > 0$

$$
\begin{aligned}
&E\left(R_t^{(n)} R_{t+s}^{(n)}\right) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} E(r_{t-i} r_t r_{t-j+s} r_{t+s}) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \left(\rho^{|i-j+s|}\rho^s + \rho^i\rho^j + \rho^{s+i}\rho^{|s-j|}\right). \qquad (1)
\end{aligned}
$$

The sum over the second and third terms of (1) is

$$
\frac{(\rho - \rho^{n+1})^2}{(1-\rho)^2} + \frac{\rho^{s+1} - \rho^{s+n+1}}{1-\rho}(1 + 2\rho + O(\rho^2)),
$$

while the sum over the first term depends on how s compares with n:

$$
\begin{aligned}
&\rho^s \sum_{i=1}^{n}\sum_{j=1}^{n} \rho^{|i-j+s|} \\
&= \begin{cases} (n-s)\rho^s + 2(n-s)\rho^{s+1} + O(\rho^{s+2}) & \text{if } n > s, \\ \rho^{s+1} + O(\rho^{s+2}) & \text{if } n = s, \\ \rho^{2s-n+1} + O(\rho^{2s-n+2}) & \text{if } n < s. \end{cases}
\end{aligned}
$$

Since

$$
\begin{aligned}
\mathrm{cor}\left(R_t^{(n)}, R_{t+s}^{(n)}\right) &= \frac{E\left(R_t^{(n)} R_{t+s}^{(n)}\right) - E\left(R_t^{(n)}\right) E\left(R_{t+s}^{(n)}\right)}{\mathrm{std}\left(R_t^{(n)}\right) \mathrm{std}\left(R_{t+s}^{(n)}\right)}, \\
E\left(R_t^{(n)}\right) &= E\left(R_{t+s}^{(n)}\right) \approx \rho, \\
\mathrm{std}\left(R_t^{(n)}\right) &= \mathrm{std}\left(R_{t+s}^{(n)}\right) \approx \sqrt{n},
\end{aligned}
$$

putting the above all together and ignoring higher-order terms proves Proposition 2. □

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