

Directional and Cross-Sectional Risk Premia: Implications for Your Portfolio

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Abstract

In recent years there has been a proliferation of alternative risk premia strategies taking advantage of well-understood empirical market inefficiencies in order to add alternative sources of return to a traditional portfolio of equities and bonds. These market inefficiencies are often referred to as factors. In this study we examine the effects of trading three generic factors – Momentum, Value, and Carry – in either a directional or cross-sectional portfolio construction framework. We demonstrate how the choice of cross-sectional or directional factor portfolio construction can affect the portfolio's leverage and transactions costs, market beta risk, and equity tail risk.

Keywords

Factor Investing; Risk Premia; Portfolio Construction

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1. Introduction

In 1976, Stephen Ross published his seminal paper *The Arbitrage Theory of Capital Asset Pricing*, in which he described a model for asset returns based on multiple return drivers called “factors” [Ross, 1976]. Before Ross, the prevailing model for describing returns was the Capital Asset Pricing Model (CAPM), in which there was only a single factor, the rather ambiguously defined “market factor” [Sharpe, 1964]. Ross opened up new avenues for understanding the drivers of returns, and it eventually led to the development of the well-known Fama-French three-factor model, in which stock returns were driven by 1) the market, 2) firm size, and 3) book value [Fama and French, 1993]. Carhart later extended this model with a fourth factor, momentum [Carhart, 1997].

Until recently, the factor concept was largely an academic construct, but it is now becoming common for institutions to invest in these factor portfolios – sometimes called style risk premia portfolios – on global markets across multiple asset classes. Despite the growth in popularity of these strategies, one often overlooked consideration is whether the factors should be traded cross-sectionally or directionally.

In factor strategies, each underlying asset is defined as having a signal or loading for each of several factors. For a given factor signal, a cross-sectional portfolio construction framework generally trades a factor by buying those assets with high signals in a given group or sector (the cross-section), and selling those assets with low signals. The goal is to essentially hedge out the sector or asset-class bench-

mark risk, resulting in a portfolio that is market-neutral. In contrast, a directional strategy trades each asset based solely on its own signal: if the signal is positive (negative), the asset is held long (sold short). Thus, a cross-sectional construction is a relative strategy, while a directional construction is an absolute strategy.

In this paper, we examine the implications of using cross-sectional and directional portfolio construction approaches on the same set of underlying Momentum, Value, and Carry signals. We find that the choice of portfolio construction has important consequences for leverage and the resulting transactions costs, market beta risk, and performance in very poor equity markets.

2. Data and Methodology

We construct backtests of Momentum, Value, and Carry portfolios across four sectors or asset classes: commodities, equities, fixed income, and foreign exchange. We use futures or forwards for each of these markets. Table 1 shows the list of markets in each sector.

For each market, we start by constructing the raw Momentum, Value, and Carry signals. For market i , the raw Momentum signal Z_i^{Mom} is the past 12 month total return divided by the daily standard deviation of that market's returns over the past 12 months. The raw Value signal Z_i^{Val} is the past five year return excluding the most recent one year, divided by the volatility over that period. In this way, the signals are all essentially z -scores.

Commodities	Equities	Fixed Income	FX
Crude Oil	Nikkei	JGB 10yr	US Dollar Index
Cotton #2	DAX	Aus. 10yr	Canadian Dollar
Natural Gas	Euro Stoxx 50	Euro Bund (10yr)	Euro
Coffee	CAC 40	Euro Bobl (5yr)	Swiss Franc
Heating Oil	FTSE 100	Euro Schatz (2yr)	British Pound
London Cocoa	S&P 500	Long Gilt (10yr)	Japanese Yen
Sugar #11	ASX SPI 200	US 30yr	Australian Dollar
Corn	OMX 30	US 10yr	New Zealand Dollar
Soybeans	AEX	US 5yr	Norwegian Krone
Wheat	S&P TSX 60	US 2yr	Swedish Krona
Copper		CGB 10yr	
Zinc			
Aluminum			
Gold			
Silver			

Table 1. Markets used in construction of the signals.

Note: For Z_i^{Val} , we use spot prices (equities and FX), nearby futures prices (commodities) and yields (bonds). When using yields, the sign of the Value signal must be reversed.

The raw Carry signal Z_i^{Carry} is the asset's estimated yield less financing cost y_i divided by its return volatility over the last 12 months. For estimating y_i , we use roll yield (commodities), earnings yield less local 12 month LIBOR (equities), yield less local 12 month LIBOR (bonds), and local 12 month LIBOR less US 12 month LIBOR (FX).

$$Z_i^{Mom} = \frac{r_i(t - 12m, t)}{\sigma_i(t - 12m, t)}$$

$$Z_i^{Val} = -\frac{r_i(t - 60m, t - 12m)}{\sigma_i(t - 60m, t - 12m)}$$

$$Z_i^{Carry} = \frac{y_i(t)}{\sigma_i(t - 12m, t)}$$

Our goal is to create simple strategies that capture differences between cross-sectional and directional portfolios. To this end; we created two sets of three portfolios for the Momentum, Value, and Carry signals respectively:

1. A sector cross-sectional set SXS, such that within each sector, signals are centered by removing the sector average signal. The centered signals are then inverse volatility weighted to form long-short portfolio weights for each sector. Sector portfolios are combined using simple inverse volatility weighting. The final portfolio is then leveraged to achieve 10% targeted annual volatility.

2. A directional set DIR, such that for each market the portfolio weight is proportional to the inverse volatility weighted scaled signal, creating one global portfolio. This portfolio is then leveraged to achieve 10% targeted annual volatility.

Next, we combine each set of Momentum, Value, and Carry portfolios into an over-arching SXS or DIR strategy using inverse volatility weighting. These strategy portfolios are then leveraged to achieve 10% targeted volatility.

3. Backtest

We backtest the portfolio strategies for Momentum, Value, and Carry signals, as well as the combined portfolios. The portfolios are rebalanced daily. We record returns both gross and net of assumed transactions costs, where transaction costs are assumed to be 1 bp (0.01%) of notional value for all markets.

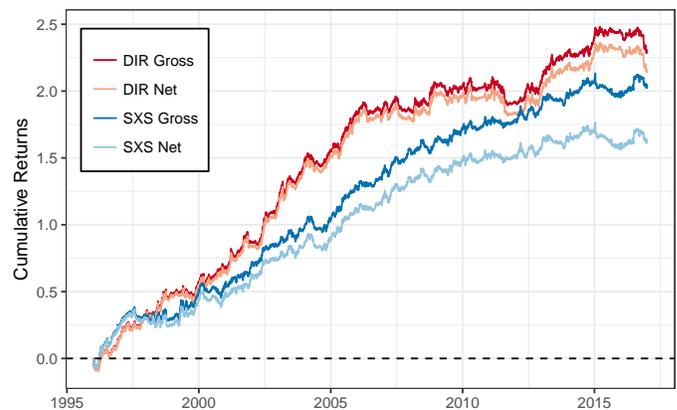


Figure 1. Gross and net cumulative returns for the combined portfolios.

	SXS	DIR
Ann. Return (Gross)	9.38%	10.59%
Ann. Return (Net)	7.48%	9.93%
Ann. Vol	10.07%	10.56%
Gross Sharpe	0.93	1.00
Net Sharpe	0.74	0.94
Max Drawdown	-18.28%	-21.47%
S&P 500 Correlation	1.47%	-9.59%

Table 2. Return statistics for the combined strategies.

Figure 1 shows the cumulative returns for the combined SXS and DIR strategies. Table 2 gives return statistics for the combined strategies, while Table 3 shows return statistics for each strategy broken out by factor.

	Mom SXS	DIR	Value SXS	DIR	Carry SXS	DIR
Ann. Return (Gross)	5.56%	9.81%	3.56%	-1.86%	8.87%	11.78%
Ann. Return (Net)	3.49%	9.15%	2.22%	-2.26%	7.04%	11.22%
Ann. Vol	10.06%	10.33%	9.95%	10.17%	9.95%	10.06%
Gross Sharpe	0.55	0.95	0.36	-0.18	0.89	1.17
Net Sharpe	0.35	0.89	0.22	-0.22	0.71	1.12
Max Drawdown	-23.12%	-19.15%	-22.15%	-76.34%	-20.25%	-29.79%
S&P 500 Correlation	-3.41%	-4.59%	0.08%	-26.98%	9.25%	19.22%

Table 3. Return statistics for the individual strategies.

In the backtest, the DIR strategy out-performed SXS on a combined basis, as well as for the Momentum and Carry factors. However, the SXS construction performed better for Value. This is not surprising. Using the negative of the past five year price return as a value indicator means that a directional portfolio would almost always be short stocks and frequently short fixed income as well.

4. Leverage and Transactions Costs

One of the first things that stands out from Figure 1, Table 2, and Table 3 is that SXS demonstrates a much wider gap between gross- and net-of-transaction costs returns than DIR. The main reason for this difference is that SXS strategies typically use more leverage than DIR strategies use. Because an SXS portfolio attempts to hedge out sector-benchmark risk, it ought to have lower volatility than a DIR portfolio on an unlevered basis. Consequently, SXS will require more leverage to reach the same level of volatility as DIR. This, in turn, drives higher transaction costs, since in practice transactions costs scale linearly or higher with leverage.

Factor	Portfolio	Avg. Leverage	Avg. T-Costs (Ann.)
Mom	SXS	0.75	2.08%
	DIR	0.35	0.66%
	Difference	0.40	1.42%
Value	SXS	0.89	1.46%
	DIR	0.37	0.44%
	Difference	0.51	1.02%
Carry	SXS	0.86	1.82%
	DIR	0.46	0.56%
	Difference	0.40	1.26%
Combined	SXS	0.87	1.89%
	DIR	0.46	0.66%
	Difference	0.40	1.23%

Table 4. Volatility-adjusted leverage and transaction costs

This is illustrated in Table 4.¹ On average, SXS leverage is roughly twice that of DIR, but transactions costs for SXS are even higher because the SXS strategy must be constantly re-balanced back to market-neutrality. This causes additional transaction costs not captured by leverage alone.

5. Correlations

One of the more common reasons for preferring an SXS construction is its perceived beta-neutrality. But we find that the market-neutrality of SXS is only true theoretically. In reality, it is very difficult to construct a truly beta-neutral portfolio. Figure 2 shows the rolling six-month correlation of the S&P 500 to the SXS and DIR strategies, respectively. The SXS strategy still has a tendency to have large movements away from zero.

As we demonstrated in Table 2, both SXS and DIR strategies tend to have rather low historical correlation to the S&P. But they earn these correlations differently: DIR tends to swing between extremely positive and extremely negative correlation as the sector-level average signal in a given equity factor switches between long or short global stocks, with the long-run effect that the high and low correlation periods roughly cancel each other out. In contrast, SXS attempts to hedge out the sector-level beta exposure on an ex-ante basis through portfolio construction, with the effect that actual realized beta to the S&P moves in a somewhat tighter range around zero.

6. Equity Tail Risks

We find that DIR Momentum tends to perform particularly well during very weak equity markets, as shown in Figure 3. In contrast, SXS Momentum performance doesn't appear to be abnormally strong or weak during either extremely strong or weak equity periods.

¹Volatility-Adjusted Leverage is defined as the sum of the 95% value-at-risks of the positions, based on the assumption of a Normal distribution: $L = \sum_{i=1}^n |w_i| 1.65\sigma_i$

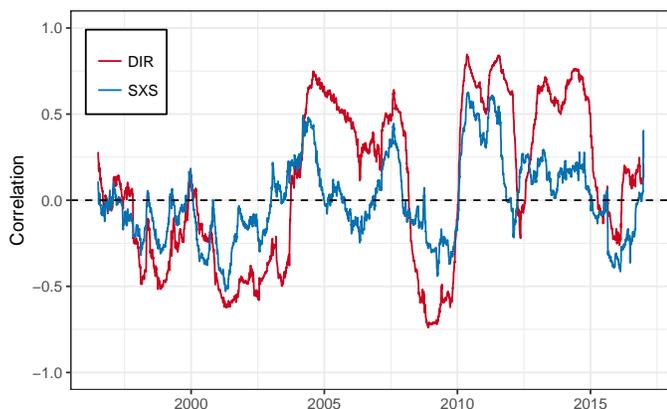


Figure 2. Rolling 6-month correlation between the momentum factors and the S&P 500.

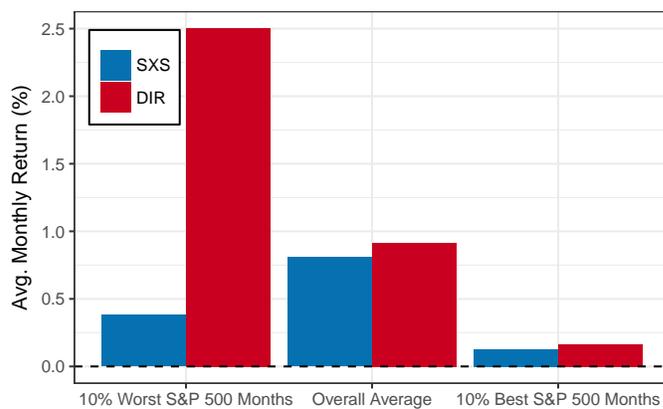


Figure 4. Performance (gross) in best and worst S&P 500 months for the combined portfolio of factors.

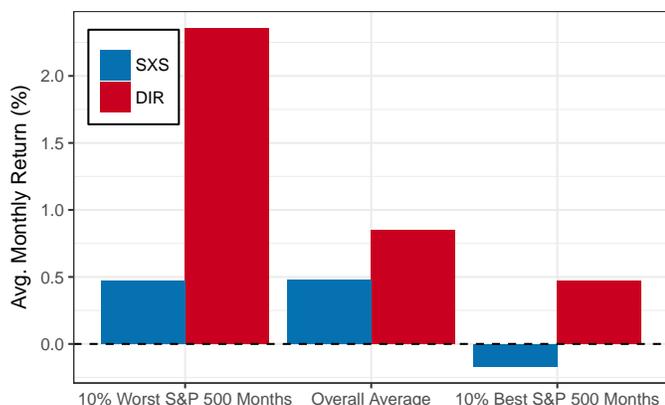


Figure 3. Performance (gross) in best and worst S&P 500 months for the momentum factor.

A large part of the strong DIR Momentum performance in weak equity markets can be explained by the ability to have large directional short positions in stocks. Since SXS attempts to remain net neutral to a sector or asset class, it does not exhibit this potential benefit.

Figure 4 shows the same results, but for the combined strategies. Even at a combined level, we see that DIR strategies show significantly stronger performance in poor equity markets.

It is tempting to argue that an investor with a large existing equity allocation would prefer an SXS strategy because of its built-in neutrality – at least in theory – to global equities. But here we see that the market-neutral nature of SXS construction may in some instances actually be *undesirable* for such an investor. While an SXS strategy might in theory be less likely to *add* to the portfolio’s equity exposure, it would also be less likely to *subtract* from it when it might be advantageous to do so.

7. Conclusion

We conducted a backtest of Momentum, Value, and Carry signals using both cross-sectional (SXS) and directional (DIR) portfolio construction. We find that the DIR portfolio outperformed the SXS portfolio. We then discussed three main distinctions between SXS and DIR portfolio construction. First, SXS strategies generally have higher leverage, and therefore higher transaction costs, than DIR strategies. Second, while SXS strategies attempt to maintain beta-neutrality, this actual real-world effect is small, with both SXS and DIR strategies showing significant equity beta over time. Finally, the tendency of SXS portfolios to hedge out asset-class or sector-level beta effects can come with a cost, since DIR strategies have the ability to be short an asset class or sector at advantageous times, while SXS strategies are less likely to exhibit this kind of behavior.

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