Contemplating Correlation

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Abstract
Correlation is widely used as a measure of the expected efficacy of portfolio diversifiers, despite the fact that it has many limitations. This note examines its shortfalls when used as a single measure and suggests supplementary methods of assessing the expected benefit that diversifying portfolio components can bring, specifically with regards to preventing or softening losses in tail scenarios. Being aware of the 'type' of diversification one seeks, an investor can purposefully choose portfolio components that fulfill the desired role. While there may be no single perfectly reliable tail loss diversifier - other than paying a significant premium for insurance in the form of options - a combination of a variety of well-chosen diversifiers can offer downside protection whilst still achieving a positive expected return.

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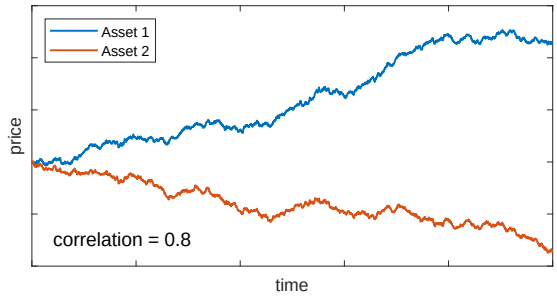
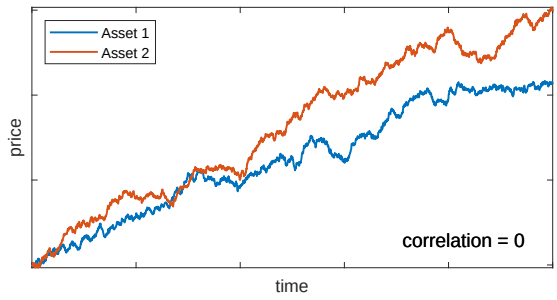
1. Introduction

It is not uncommon for asset owners to feel disappointed with their portfolio diversifiers due to unforeseen correlation with risk assets on the downside. Understanding nuances in the nature of 'correlation', using additional measures when assessing a potential diversifier, modifying strategies to be more effective diversifiers, as well as taking a more 'diversified' approach to diversification all can help to improve portfolio diversification overall. In this paper we clarify some of the common misconceptions around correlation, and look further at what can be done to help in selecting more effective portfolio diversifiers.



(a) Two highly correlated series.



(b) Two uncorrelated series.

Figure 1. Plots of two price series that are a) 80% correlated or b) uncorrelated, yet move in different or the same directions, due to different or similar average returns.

2. Correlation Pitfalls

Mathematically, correlation measures the statistical relationship between two variables. While it does not imply dependence, it indicates to which degree two variables tend to move 'in sync' with each other. Correlation does not take into account the average move of each variable, which can lead to counterintuitive correlations at times, as illustrated in Figure 1.

Figure 1(a) shows two highly correlated variables with different means, with one series trending up and the other down. The variables in Figure 1(b), however, trend in the same direction, but have zero correlation: when two such assets are combined, fluctuations around the mean return are washed out, increasing the Sharpe Ratio. This effect becomes stronger the more uncorrelated assets are aggregated, see Tricker and Mitchell (2017). (Note that combining assets with positive average returns 'obviously' gives an overall positive return. Interestingly, we may want to add an asset with *negative* average return to a portfolio, if its addition enhances performance due to low correlation.)

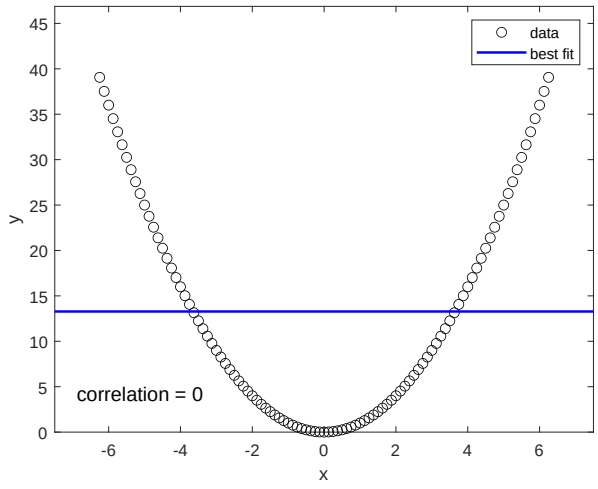


Figure 2. Plotting $y = x^2$ for a range of values of x. While y is fully determined by x, the *linear* correlation between the two, measured by the slope of the line of best fit, is zero.

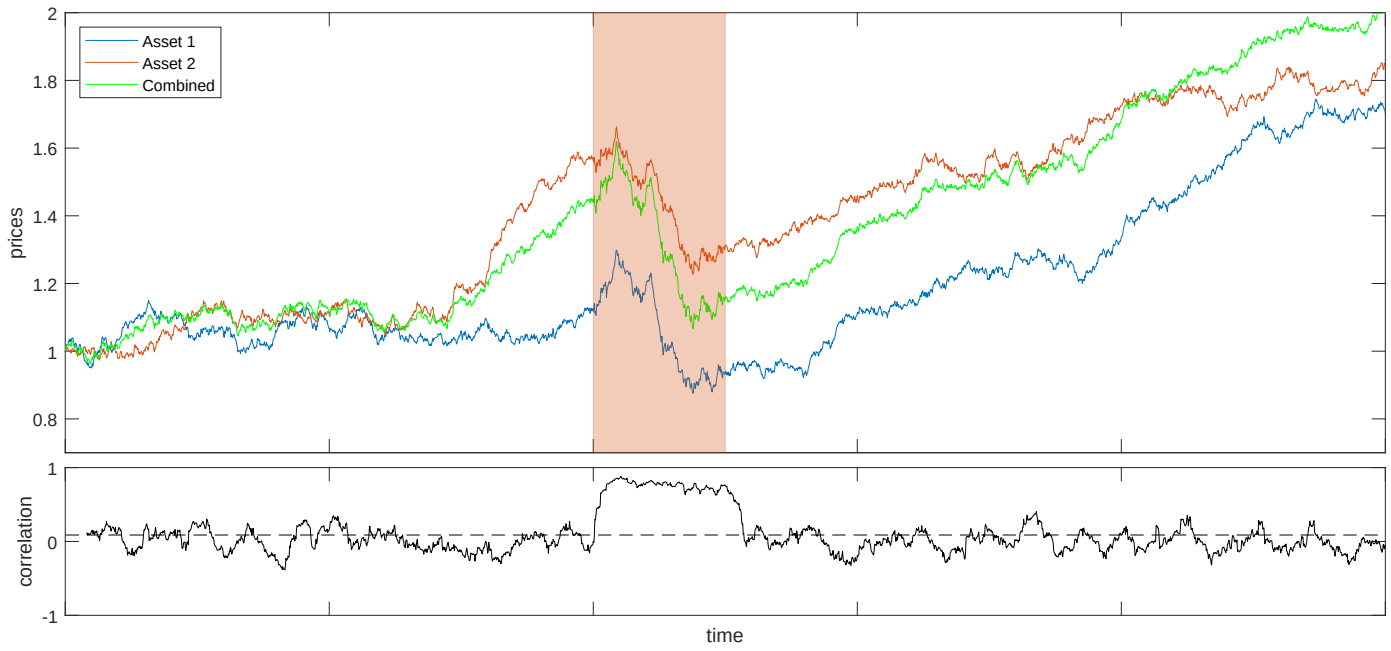


Figure 3. Two simulated price series that exhibit significant correlation of returns during tail events, and no correlation during other periods. We also include the 40-day rolling correlation, which can be seen to even be significantly negative at times.

Furthermore, the most common correlation measure is designed to detect *linear* relationships, which can lead to further issues. For an example, Figure 2 shows a plot of a simple non-linear (and non-monotonic) function $y = x^2$. Here the value of x fully determines the value of y (and vice versa) and the two variables are thus dependent. Yet, because the way we measure correlation here only detects linear relationships, we find zero correlation between x and y even though the value of y is entirely dependent on the value of x.

Even in the stylized examples above we can see that correlation can be counterintuitive or even misleading, and should not be relied upon blindly. In reality, correlations change over time, sometimes slowly and sometimes abruptly, see Tricker and Mitchell (2017). In the next section we therefore look at an even bigger problem for many allocators – conditional correlation.

3. Conditional Correlation

Sometimes asset owners face the worst of all worlds - portfolio diversifiers that are only uncorrelated with their core portfolio in normal market conditions, but become correlated when most needed, when the core is under stress. We give a simple example of this in Figure 3, using two simulated price series; we also show the rolling correlation between the assets. While the zero overall correlation, and existence of negative correlation at times, would suggest that combining these two assets would help build a diversified portfolio, the opposite is in fact true. The high positive tail correlation during market distress would eliminate any protection an investor might have believed to have had.[1]

A more insightful measure when seeking diversification in specific market environments is the *conditional correlation* of one asset to the other, conditioned on some 'situation of interest'. In our example this situation of interest would be a notable *market downturn* over a relatively short period of time. To really diversify our portfolio, we need to avoid positive conditional correlation in such scenarios, aiming for zero or negative correlation instead.

The limitation of this type of conditioning is that such instances occur infrequently, and so there are few such periods over which to measure correlation. Another option is conditioning correlation on all negative market periods of any significance, for example, all months where the market falls by 3% or more – this gives a potentially larger set of instances over which correlation may be measured, and so likely gives a more robust assessment of correlation in negative market conditions. Adding an asset to a portfolio that consistently has positive returns when the existing portfolio has negative returns, i.e. exploiting negative conditional correlation, would be highly beneficial to downside control, but of course the behavior of this diversifier in 'up markets' also needs to be assessed.

In Table 1 below we look at the monthly correlation and conditional correlation to the S&P 500 of a range of assets that may benefit a portfolio during an equity market decline or in more commonplace negative market periods. The data spans the period January 2000 through December 2020. We see that Treasuries, Gold, CHF or puts have low or negative correlation to equities, with this holding up on average in crises, so these assets provide good portfolio diversification then. We also see

[1] Figure 3 also brings back the point raised earlier about correlation not implying anything about average returns. The aggregate return of a portfolio of two assets that trend down is negative, regardless of their correlation. Diversification via correlation reduces variance or uncertainty of returns, but does not affect the mean.

that Trend, Macro/CTA strategies (HFRXM) and Equity Market Neutral strategies (HFRXEMN) seem all to be uncorrelated to the S&P 500 portfolio and so act as good diversifiers overall, but there is a big difference when we look at conditional correlation during negative markets. The correlation of Equity Market Neutral strategies actually increases in down months (making these less suitable diversifiers), whereas Trend and Macro/CTA become negatively correlated (becoming better diversifiers). A similar picture emerges when contrasting the Cross-Asset Risk Premia and Trend Indices published by Bloomberg GSAM. Analysis of correlation alone would make an investor indifferent between some of these, but through analysis of conditional correlation, we see that trend-following and Macro/CTA strategies are more effective diversifiers than, for example, Equity Market Neutral and Risk Premia strategies in down markets.[2]

Asset	Overall	Down	Down 3%
US Treasuries	-0.34	-0.21	0.04
Gold	0.04	0.02	0.28
CHF	0.14	0.10	0.13
Put Buy	-0.84	-0.79	-0.70
Trend*	-0.09	-0.32	-0.33
Equity Market Neutral**	0.06	0.14	0.10
Macro/CTA**	0.07	-0.13	-0.05
X-Asset Risk Premia***	0.11	0.21	0.39
X-Asset Trend***	-0.30	-0.46	-0.52

Table 1. Monthly correlations of different assets to the S&P 500 when measured overall, when the index is down, and when it is down at least 3%. Data ranges from January 2000 to December 2020. * SG Trend Index, ** HFRX, *** BBG GSAM.

4. Companion Papers: Improving Portfolio Diversification - A Multi-Asset, Multi-Strategy Approach

As we have seen above, some assets are disproportionately better diversifiers than others in negative market conditions compared to overall, but we have not addressed the trade-off between diversification in negative markets and the overall return of the portfolio. DeWoskin et al. (2020a) explore these diversifiers with a focus on both their long-term and crisis performance. Amongst the *passive* diversifiers, bonds are usually added to portfolios when crisis protection is sought; they have historically provided consistent returns and have displayed negative correlation to the equity markets over much of the last couple of decades. However, as outlined in Fan and Mitchell (2017), there is no compelling reason why equity-bond correlation should be negative, and, indeed, positive correlations have been persistent over much of the last century, as well as recently.[3] In addition, as discussed in DeWoskin et al. (2020b), with bond yields currently very low, it is unlikely bonds will provide significant downside protection going forward. *Active* diversifiers aim to provide protection by design rather than luck. Trend-following and put options, for example, can explicitly manage diversification in a way that many passive investments cannot. DeWoskin et al. (2021) outline how assets with different diversification characteristics may be combined to obtain a trade-off between crisis protection and long-term return.

5. Conclusion and Outlook

In this note we have shown that correlation as a single efficacy measure of a diversifier may mislead: conditional correlation may reveal that strategies that are similarly correlated to equity markets overall behave very differently in down-markets. One way of improving portfolio diversification is to invest in a variety of diversifying assets and strategies with desirable *conditional* behavior. Another way can be to build portfolios that control correlation through the capping of exposure to an underlying market, and we will consider this approach in a subsequent paper.

References

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[2] Investment strategies such as trend-following display convexity of returns, being able to generate positive returns when markets suffer. Interestingly, an 'ideal' convex return profile for a diversifying asset would look like Figure 2. We have seen that the correlation in this stylized case is zero, illustrating why trend-following displays such low overall market correlation.

[3] This illustrates the *non-transitivity* of correlation: equity-bond correlation has recently shot up to become positive, yet the correlations between equities and oil, and bonds and oil, for example, have remained postive and negative, respectively.